Traditions and Innovations in Baltic and Nordic Mathematics Education: A Methodological Guide for Higher Education

Publication Information

This guidebook was developed within the framework of the **Nordplus Higher Education** project **"Traditions and Innovations in Baltic and Nordic Mathematics Education / 2025"** (Project No. **NPHE-2025/10040**).

Instrument: Nordplus Higher Education Programme

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Dissemination Level: Public

Date of Publication: 21 July 2026

Publisher: Daugavpils University

ISBN 978-9934-39-060-9

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Introduction

The European Union is currently facing a critical shortage of skilled professionals in STEM fields (Science, Technology, Engineering, and Mathematics). A key contributing factor is the stagnation in university enrollment and graduation rates within disciplines, particularly in the Baltic states. University mathematics teaching materials are often updated slowly and remain overly conceptual. They are fragmented, failing to align with the dynamic demands of the modern labor market and the practical needs of professions in engineering and natural sciences (Song, 2014)[1].
The project is set against a backdrop of significant challenges in STEM education across the participating countries. We found that Finland has the highest proportion of students enrolling in STEM fields (33.9%), while Latvia has the lowest (24.0%) (European Commission, 2026[2]; European Commission, 2025a[3]). In 2022, only 19.7% of graduates in Latvia were from STEM fields – one of the lowest shares in the EU, where the average stands at 26.6%. (Institute of Economics of the Latvian Academy of Sciences [IE LVA], 2025[4]) These figures highlight the urgent need for educational interventions that make STEM subjects, particularly mathematics, more engaging and relevant to students (Song, 2014)[1].
There is a lot of evidence, according to research, that studying mathematics is generally quite procedural and poorly fitted to using and working with mathematics in science, engineering, and other fields. Instead of gaining a relational, relevant, and creative understanding of mathematics that they may apply more broadly, students frequently learn to replicate mathematical operations in accordance with tests and exams (Ichsan et al., 2024)[5].
This project brings together higher education institutions from Latvia, Estonia, and the Åland Islands (Finland) regions that share similar population sizes and historical development contexts. This common ground provides a strong foundation for collaboratively developing solutions to enhance mathematical skills in higher education that can be effectively applied across all participating countries.

Main Goal

The goal of the project "Traditions and Innovations in Baltic and Nordic Mathematics Education/2025" is to bridge the gap between theoretical mathematics education and its practical, real-world application by modernizing teaching materials and methodologies. The project aims to:

- Develop innovative teaching materials for higher education mathematics;
- Promote up-to-date teaching methods in mathematics for higher education;
- Apply mathematical knowledge in real-life and professional settings;
- Facilitate the systematic exchange of experiences and best practices between partner institutions.

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[1] Song, Y., & Kong, S. C. (2014). Going beyond textbooks: A study on seamless science inquiry in an upper primary class. *Educational Media International*, *51*(3), 226–236. https://doi.org/10.1080/09523987.2014.968450

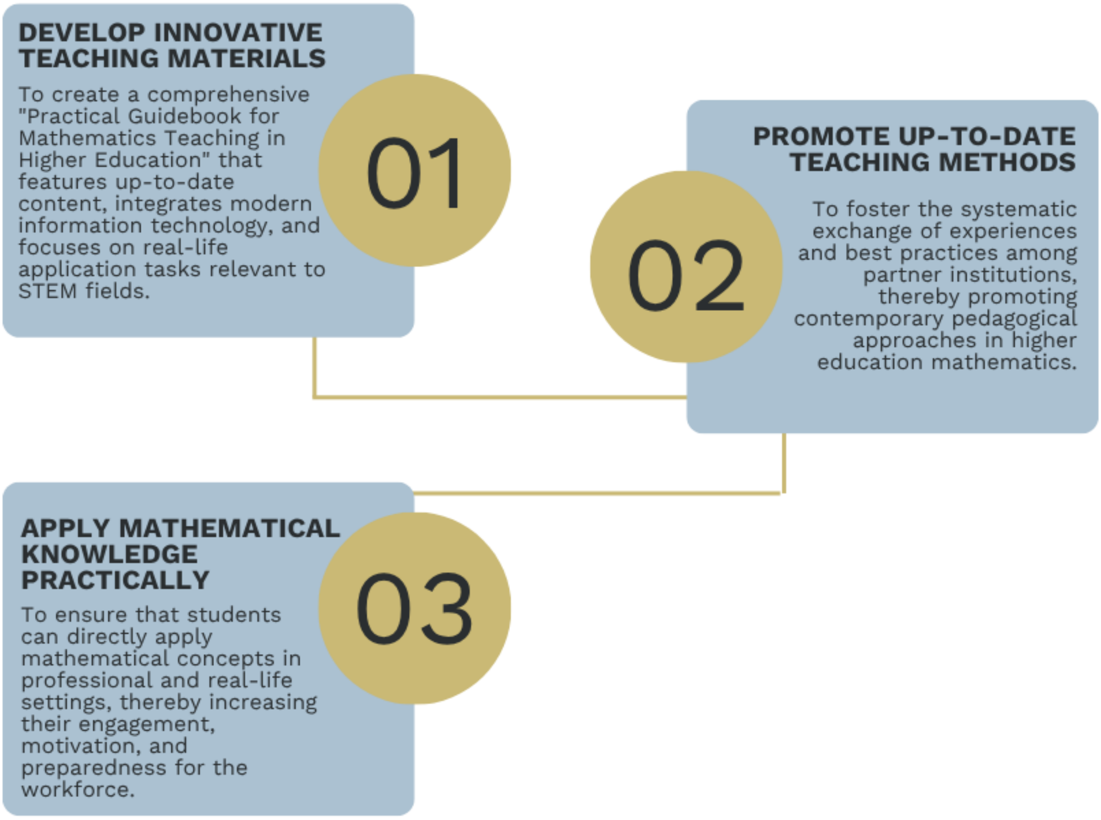
[2] European Commission. (2026, March). *Boosting STEM participation in tertiary education: a persistent challenge*. Education Chart of the Month. Directorate-General for Education, Youth, Sport and Culture. https://education.ec.europa.eu/resources-and-tools/data-and-analysis-on-education-and-skills/key-trends-in-education/stem-in-tertiary-education

[3] European Commission. (2025a). *Education and training monitor 2025: Country report: Latvia*. Publications Office of the European Union. https://op.europa.eu/webpub/eac/education-and-training-monitor/en/country-reports/latvia.html

[4] Institute of Economics of the Latvian Academy of Sciences. (2025, September 24). *Latvia monthly briefing: STEM youth development drives fintech growth: Latvia case study*. China-CEE Institute. https://china-cee.eu/2025/09/24/latvia-monthly-briefing-stem-youth-development-drives-fintech-growth-latvia-case-study/

[5] Ichsan, I., Mulyati, M., Duma, S. Y., & Nirfayanti, N. (2024). The effectiveness of mathematics learning with the STEAM approach in improving students' critical thinking skills. *Aksioma Education Journal*, *1*(3), 1–16. https://doi.org/10.62872/kdepgn33

Figure 1: Three core pillars to achieve the project goal



The expected outcome of the three pillars is that by pillar 1, enhanced teaching quality and improved teachers' well-being, educators will save time preparing for lecture materials. Pillar 2 will improve student motivation and achievement, as mathematics teaching will be up-to-date, connected to real-life applications, and supported by a comprehensive collection of advanced mathematics tasks. Pillar 3, the “Practical Guidebook for Mathematics Teaching in Higher Education,” will contain up-to-date teaching resources and real-life application tasks.

Table 1
Guidebook Structure and Content

Based on the analysis of course descriptions and identified needs, the guidebook is structured to provide:

Component	Description
Theory	Clear explanations of mathematical concepts with visual aids
Tasks at Different Difficulty Levels	Progressive exercises from basic to advanced
Practical Applications	Real-world problems from various professional fields

Applications Included in the Guidebook

Table 2
Vector applications

Application Area	Vector Concepts Used
Physics and Engineering	Vector addition, subtraction, scalar multiplication; dot product and cross product; analysis of forces, velocity, and acceleration
Robotics and Mechanics	Position vectors, torque (cross product), kinematics and dynamics modelling
Navigation and GPS	Vector addition, vector components, direction angles, ground speed and course calculation
Computer Graphics and Technology	Position vectors, transformations (rotation, translation, scaling), lighting calculations (dot product)

Table 3
Linear algebra applications

Application Area	Mathematical Concepts Used
Business and manufacturing	Matrix multiplication, revenue calculation
Circuit analysis	Systems of linear equations
Production planning	Matrices, input-output models
Manufacturing cost analysis	Matrix multiplication
Computer graphics	Matrix transformations
Image processing	Matrix operations

To ensure the development of relevant and up-to-date teaching materials, an analysis of existing course descriptions was conducted at the partner institutions. This analysis examined the literature used in mathematics courses, identifying strengths, gaps, and areas for improvement.

Daugavpils University
The analysis of nine course descriptions reveals that the recommended literature is generally well aligned with course objectives and supports both theoretical understanding and practical skill development.

Table 4
Daugavpils University math books

Finding	Description
Literature Base	Multilingual, combining Latvian and English sources
Accessibility	High, due to the inclusion of open educational resources
Strength	Literature is generally well aligned with course objectives

Challenge	Partially outdated, especially in software-related and applied contexts

Riga Technical University
The analysis of 13 course descriptions from RTU revealed a total of 34 unique literature sources.

Table 5
Riga Technical University math books

Finding	Description
Literature Base	Predominantly Latvian (73.5%), with English (23.5%) and minimal representation of other languages (3.0%); includes comprehensive textbooks, problem collections, and methodological materials
Accessibility	Accessible for foundational learning through university library and OER; language barriers limit access for international students
Strength	Adequate for foundational learning; well aligned with course objectives; includes problem collections and methodological materials
Challenge	Heavy reliance on older sources limits adequacy for advanced/research-oriented study; limited English-language resources restrict international accessibility

Åland University of Applied Sciences
The analysis of 18 course descriptions from Åland University of Applied Sciences reveals a comprehensive mathematics curriculum designed specifically for nautical engineering, information technology, and business programs.

Table 6
Åland University of Applied Sciences math books

Finding	Description
Literature Base	Multilingual, combining Swedish and English sources, with a strong emphasis on Scandinavian textbooks (Mathematics 5000+ series); includes Croft's Engineering Mathematics, Körner & Wahlgren's Statistical Methods, and Fajardo's Spherical Trigonometry
Accessibility	High, with extensive use of lecture notes, exercise materials, and supplementary resources distributed by teachers; several courses incorporate open educational resources
Strength	Strong alignment between course content and professional applications, particularly in nautical engineering; progressive course structure; integration of Excel and MATLAB
Challenge	Heavy reliance on Swedish-language textbooks limits accessibility for international

TTK University of Applied Sciences
The analysis of 7 course descriptions from TTK University of Applied Sciences reveals a focused mathematics curriculum designed primarily for engineering and economics programmes.

Table 7
TTK University of Applied Sciences

Finding	Description
Literature Base	Predominantly Estonian-language sources, with limited English-language references; heavy reliance on teacher-distributed materials (lecture notes, exercise materials)
Accessibility	High, due to extensive use of lecture notes, exercise materials, and supplementary resources provided by teachers; limited English resources
Strength	Strong focus on practical application and real-world problem-solving; learning outcomes emphasize critical evaluation of results
Challenge	Limited use of up-to-date textbooks; over-reliance on teacher-generated materials may create inconsistencies

Table 8
Key Findings Summary

Finding	Implication
Outdated literature	Particularly in software-related and applied contexts; limits adequacy for advanced or research-oriented study where mathematical applications have evolved
Language barriers	Heavy reliance on local language (Latvian, Swedish, Estonian) limits accessibility for international students and exposure to international research
Limited practical focus	Need for more real-world application examples; some institutions already demonstrate strong professional orientation
OER potential	Open educational resources improve accessibility and can address outdated content; currently unevenly implemented across institutions

This guidebook is primarily intended for higher education educators who teach mathematics, particularly those at universities and applied sciences institutions in the Baltic and Nordic regions. These educators will benefit directly from the project's outputs, including innovative teaching materials, real life application tasks, and training workshops designed to help them implement new methods effectively. A second primary beneficiary group consists of students in STEM fields, who will gain access to mathematics education that is more relevant, up to date, and connected to professional practice. International students are also explicitly recognized as beneficiaries, since the teaching materials will initially be prepared in English, making them accessible to the significant number of international students enrolled at the partner universities. Beyond these direct beneficiaries, the guidebook targets STEM educators and students across the broader Baltic and Nordic regions, as well as educational networks that can ensure wide visibility within the academic community. The guidebook is collaboratively built by and for four participating institutions: Daugavpils University (the coordinating institution in Latvia), Riga Technical University (Latvia), TTK University of Applied Sciences (Estonia), and Åland University of Applied Sciences (Åland, Finland). Through dissemination via publications, conferences, workshops, and online platforms, the guidebook is also intended for other institutions and educators beyond the immediate partnership.

Inquiry-based learning and mathematics teaching

In the world that is changing, there is a need to attract people who will associate their future career in STEM. One of the eight key competence areas is therefore the provision of basic education in natural and exact sciences, with special emphasis on developing practical research skills through experiments and references to everyday life European Commission, 2025b[6]). According to the Rocard (2007[7]), the best teaching strategy in this area is inquiry-based learning (IBL).

Inquiry- based learning (IBL) in Figure 2 (Sokołowska, 2020)[8] is explained as a learning approach in which students, rather than memorizing facts, follow a scientific method to discover knowledge and solve problems through hands-on exploration, questioning, and idea exchange, while the teacher shifts from direct instruction to supporting the learning journey.

Figure 2: Inquiry-based learning description

Inquiry-Based Learning (IBL) is an active, hands-on approach where students learn by using the scientific method to explore questions and solve problems. Instead of passively memorizing facts, students discover knowledge themselves, with teachers acting as supportive guides rather than direct instructors.

A student-centered approach to teaching science and mathematics, inquiry-based mathematics education (IBME) encourages students to work in methods that are comparable to those of mathematicians and scientists. Therefore, students are urged to (Dorier & Maab, 2020)[9]

- observe phenomena and ask questions;
- look for mathematical and scientific ways to answer these questions;
- carry out experiments and systematically control variables;
- draw diagrams, calculate, look for patterns and relationships;
- make conjectures and generalizations;
- interpret and evaluate their solutions;
- communicate and discuss their solutions effectively.

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[6] European Commission. (2025b, August 21). *Key competences*. European Education Area. https://education.ec.europa.eu/focus-topics/improving-quality/key-competences

[7] Rocard, M. (2007). *Science education now: A renewed pedagogy for the future of Europe*. Office for Official Publications of the European Communities. http://ec.europa.eu/research/science-society/document_library/pdf_06/report-rocard-on-science-education_en.pdf

[8] Sokołowska, D. (Ed.). (2020). *Volume 1: Inquiry based learning to enhance teaching: Theory, tools and examples*. University of Ljubljana, Faculty of Education. https://ec.europa.eu/programmes/erasmus-plus/project-result-content/cf1a25b1-14ed-4c62-81db-1d4754bb0725/3D_VOLUME1.pdf

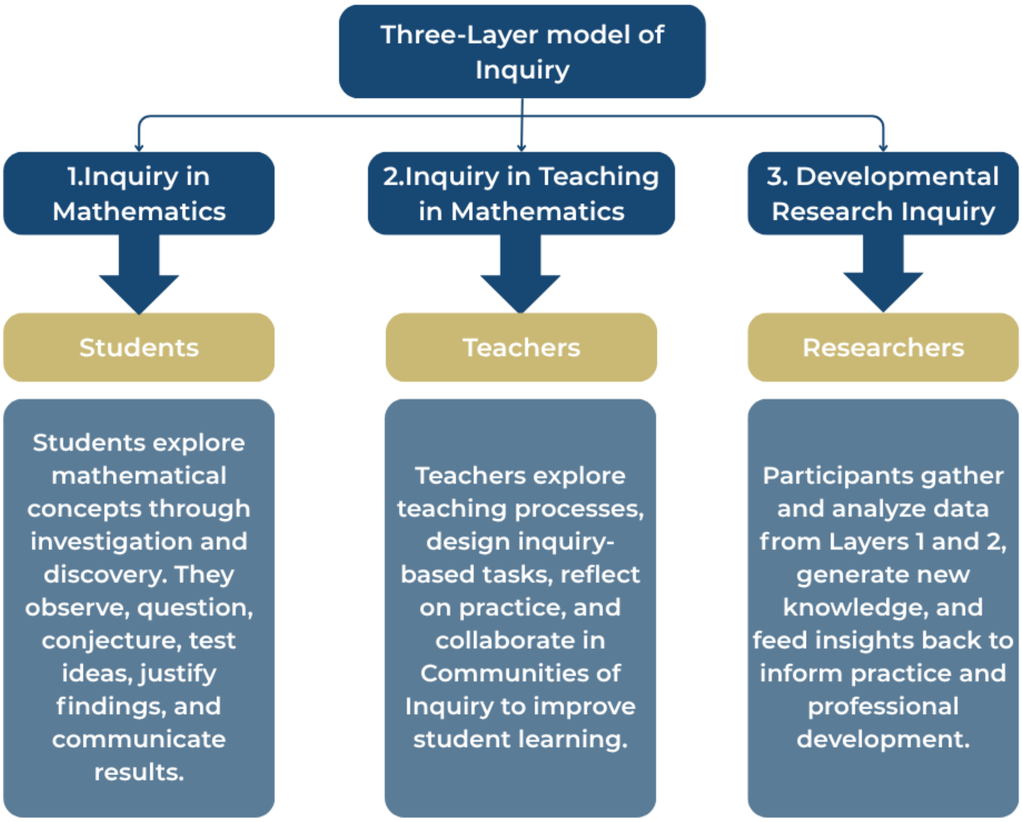
[9] Dorier, J.-L., & Maaß, K. (2020). Inquiry-based mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (2nd ed., pp. 384–388). Springer Verlag.

Unlike traditional instruction or learning by the book, the primary goal of inquiry is to deeply engage students with topics that they should learn or develop. When it comes to mathematics, inquiry-based approaches to issues and tasks enable students to engage with the subject by not only using standardized rules and procedures but also by exploring and studying concepts and processes, attempting to solve problems, and providing answers to open-ended questions.

The three-layer model of inquiry

The good example of a three-layer model of inquiry was found in Rogovchenko et al. (2021)[10], which created a detailed three-layer model of inquiry that offers the theoretical guidebook for comprehending the various interrelated levels at which inquiry-based mathematics instruction functions. This paradigm acknowledges that teachers and researchers also participate in inquiry, and that these three levels are intricately linked and reinforce one another, rather than viewing inquiry as just a student activity.

Figure 3: Three-layer model of inquiry



[10] Rogovchenko, S., Rogovchenko, Y., & Tretyakova, O. (2021). Inquiry-based tasks with modeling and conceptual questions. In I. M. Gómez-Chacón, R. Hochmuth, S. Rogovchenko, & N. Brouwer (Eds.), *Mathematics for higher education: INDRUM 2020 and PLATINUM open perspectives* (pp. 79–88). Masaryk University Press. https://doi.org/10.5817/CZ.MUNI.M210-9983-2021-7

Layer 1: Inquiry in Mathematics

In this layer, students engage actively with mathematical content through inquiry-based approaches. Instead of passively receiving information, students:

- **explore** mathematical phenomena and problems;
- **ask questions** that drive their investigation;
- **formulate hypotheses** about mathematical relationships;
- **plan and conduct investigations** using appropriate tools;
- **analyze data** and look for patterns;
- **conclude** and compare with initial hypotheses;
- **share results** with peers and engage in constructive discussion;
- **develop problems** further, opening the cycle for continued inquiry.

Key characteristics: Unstructured assignments with several possible solutions, student responsibility for their own learning, active encouragement of discussion and teamwork, and the teacher's role as a facilitator rather than an information provider are the hallmarks of this teaching methodology.

Example: Students investigating matrix multiplication by exploring real-world revenue calculations rather than simply memorizing the procedure.

Picture 1 is *adapted from linear algebra textbook, matrix multiplication application problems*

Problem 3

Two companies manufacture desk lamps, office chairs, and bookshelves. The quantity of each product manufactured per month is shown in the Table 1.

Table 1

Company	Desk lamps	Office chairs	Bookshelves
Company 1	90	150	75
Company 2	120	100	90

The unit prices of each product in euros for three consecutive months are given Table 2.

Table 2

Product	March	April	May
Desk lamps	35	36	38
Office chairs	58	60	59
Bookshelves	72	75	78

Tasks

a) Construct a matrix of product quantities for each company.
b) Construct a matrix of unit prices for the three months.
c) Compute the revenue matrix for each company for each month by multiplying the quantity matrix by the price matrix.
d) Explain the economic meaning of the resulting matrix elements.

Layer 2: Inquiry in Teaching Mathematics

Teachers examine their own pedagogies, didactics, task designs, and classroom interactions on this layer. They consider what works, what doesn't, and why. They create, implement, observe, and refine inquiry-based tasks and teaching packages.

Key characteristics: According to this model, professional development results from methodical reflection in teacher Communities of Inquiry (CoI), where teachers work together to analyse student reactions, misconceptions, and engagement in order to modify materials based on actual data from their practice.

Key questions for teachers:

- What makes a task inquiry-based?
- How do students respond to different types of inquiry tasks?
- What scaffolding do students need at different stages?
- How can I balance structured and open inquiry?

Example: Lecturers meeting regularly to discuss how to transform a traditional circuit problem into an inquiry-based task about Kirchhoff's laws, then observing student responses and refining the task.

Picture 2 *adapted from linear algebra textbook, matrix multiplication application problems*

5) An electric circuit has three loops. According to Kirchhoff's laws, the system of equations is

$$\begin{cases} 10I_1 - 2I_2 + I_3 = 5 \\ -2I_1 + 8I_2 - I_3 = 0. \\ I_1 - I_2 + 5I_3 = 10 \end{cases}$$

Find the currents I_1, I_2, and I_3.

Layer 3: Development Research Inquiry

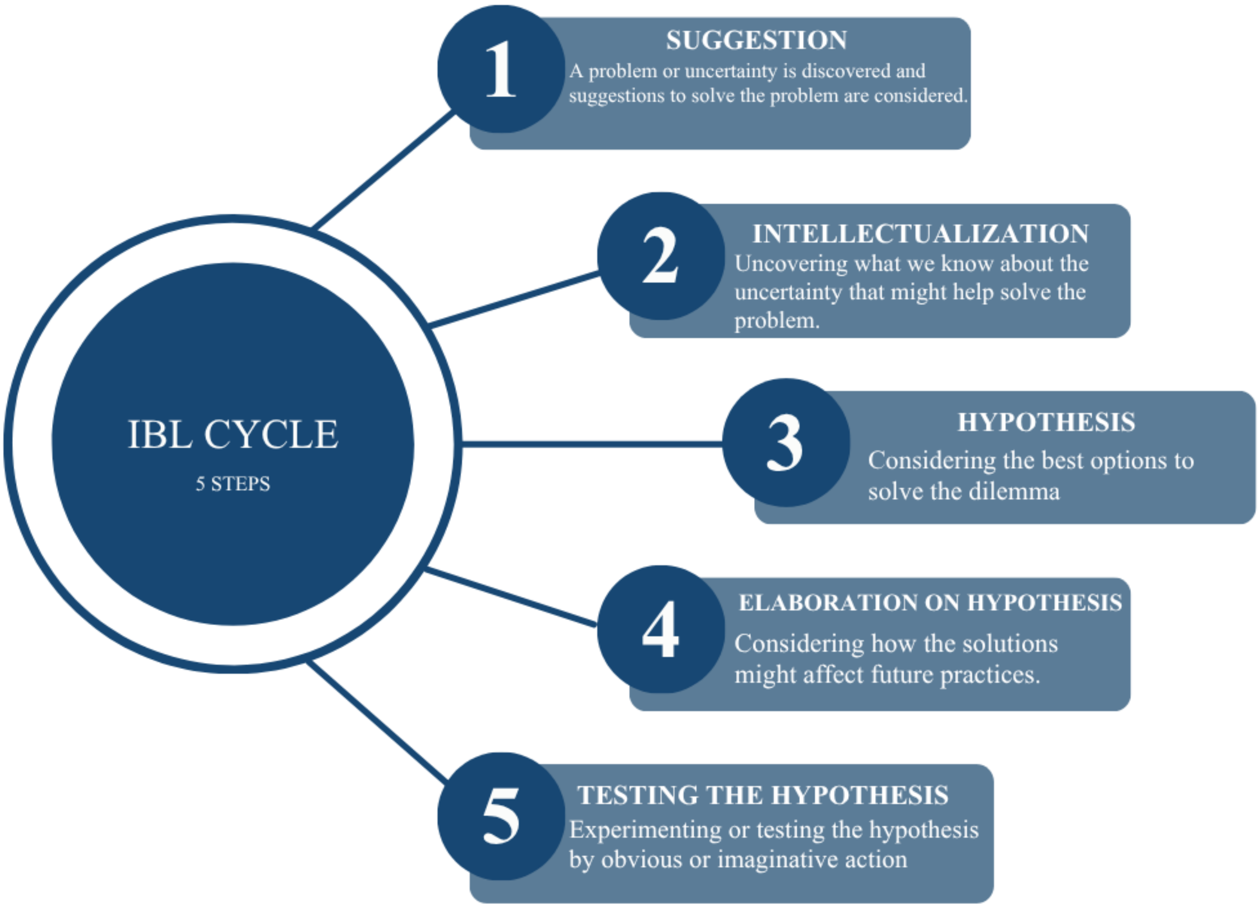
Participants in this layer, which includes teachers, researchers, and math educators, take a step back to examine the entire developmental process. They collect and evaluate information from Layers 1 and 2, produce fresh insights regarding inquiry-based practice, and contribute to professional growth and instruction.

Key characteristics: In order to analyse and detect trends, problems, and transferable principles, this method focuses on systematic data collecting, which includes observations, questionnaires, and interviews. The results are then shared with the larger academic community to develop theory that guides future practice.

IBL cycle

IBL cycle is an old process that consists of Dewey's five steps[11] (Figure 3), originally "pattern inquiry". Bruner[12] (1960) further developed the idea into a learning cycle that mimics the scientific research process. The cycle is closed (complete unit) but also open for extending the problem. The author represents 9 stage IBL cycle first one being setting the scene and generating idea, second formulating an inquiry question (open-ended, not yes/no), third making hypotheses, fourth planning investigation (roles, materials, action plan – may be trial-and-error for beginners), fifth carrying out the investigation, sixth data analysis, seventh concluding (compare with hypotheses), eight sharing results (present, compare, ask constructive questions), and ninth developing the problem (opening the cycle). The cycle is adaptable; phases can be repeated, omitted, or modified. The objective could be soft competencies, practical skills, or content understanding.

Figure 4: IBL cycle 5 steps



[11] Dewey, J. (1989). How we think. In J. A. Boydston (Ed.), John Dewey: The later works, 1925–1953 (Vol. 8, pp. 105–352). Southern Illinois University Press. (Original work published 1933)

[12] J. Bruner, (1960), *The Process of Education*, Harvard University Press, Cambridge Massachusetts, London England (1960),p.

Levels of inquiry-based learning

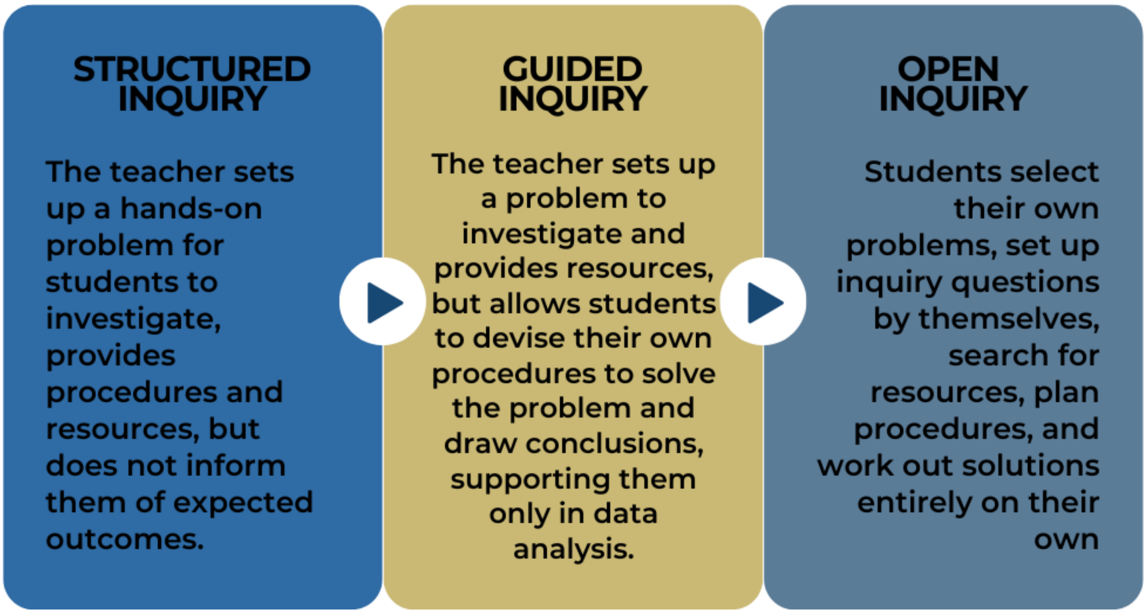
The teacher and students share responsibility for completing each stage of the IBL cycle. The first stage, which involves setting the atmosphere and coming up with ideas, is the only phase that the teacher and students always go through together. Students become more independent and proficient in using the IBL approach the less the teacher steps in. There are three IBL levels you can see in Figure 4, the higher the level, the more independent student is from teacher and their instructions (Kirschner, Sweller, & Clark, 2006[13]; Lazonder & Harmsen, 2016[14]; Furtak et al., 2012[15]).

When using the IBL technique, a teacher should select the method's level based on the lesson's objectives and particular classroom settings. Unless the activity is an extracurricular instruction designed to boost motivation and good attitudes and is deeply ingrained in students' interests, it is not advised to begin using the IBL technique by implementing the entire cycle. In other situations, the cycle should be used gradually in accordance with the age and skill levels of the students, their prior IBL experience, and the specific scientific issues or challenges.

It is acceptable to begin work at the lowest (structured) level in earlier grades and in classrooms that have never used this method before, and to move on to the second (directed) level only after applying multiple cycles. It is advised for students who are already familiar with the approach because the highest (open) level already needs students to be highly autonomous and accustomed to inquiry procedures. However, if students are mature enough to select an inquiry topic of their own interest, open inquiry can occasionally be presented as a sort of interlude between implementations of the technique at the first two levels, boosting students' curiosity and research dedication. Without a doubt, students' ingenuity is utilized at the open level. Without a doubt, students' creativity is completely utilized at the open level, where it is also most fully developed through repeated participation in the entire IBL work cycle. However, these kinds of exercises are typically not included in the curriculum. It is important to note that studies on the IBL method's efficacy indicate that the first two levels of instruction yield the best learning outcomes (Kirschner, Sweller, & Clark, 2006[16]; Lazonder & Harmsen, 2016[17]; Furtak et al., 2012[18]).

[13] Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75–86.
[14] Lazonder, A. W., & Harmsen, R. (2016). Meta-analysis of inquiry-based learning: Effects of guidance. *Review of Educational Research, 86*(3), 681–718. https://doi.org/10.3102/0034654315627366
[15] Furtak, E. M., Seidel, T., Iverson, H., & Briggs, D. C. (2012). Experimental and quasi-experimental studies of inquiry-based science teaching: A meta-analysis. *Review of Educational Research*, *82*(3), 300–329. https://doi.org/10.3102/0034654312457206

Figure 5: IBL levels



As mentioned before, we have developed teaching materials, and in those teaching materials, you can see the 3 levels of IBL. Further, there will be examples from our teaching materials in each stage.

Structured Inquiry (Lowest level – teacher provides problem, procedures, and resources; outcome not disclosed) *(Picture 3)*

Topic: Matrix Addition and Subtraction

A teacher presents the following problem to students. The teacher provides the data, the procedures (matrix addition, subtraction, and average calculation), and the resources, but does not reveal what the results represent. Students must discover the meaning through their calculations. Data on the production of three types of agricultural products – grain, milk, and meat (in conventional units) – in two farms for the years 2023 and 2024 are given in the form of matrices.

Picture 3 *is adapted from linear algebra textbook, matrix multiplication application problems*

Problem 1

Data on the production of three types of agricultural products – grain, milk, and meat (in conventional units) – in two farms for the years 2023 and 2024 are given in the form of matrices:

$$A_{2023} = \begin{pmatrix} 2053 & 571 & 340 \\ 2679 & 453 & 694 \end{pmatrix}, A_{2024} = \begin{pmatrix} 2745 & 644 & 800 \\ 2900 & 451 & 373 \end{pmatrix}.$$

Find:

a) the total volume of products produced over the two years (2023 and 2024);
b) the growth in production volumes in 2023 compared to 2024, by product types and by farming enterprises;
c) the matrix of average annual production of products.

You should also explain the economic meaning of the elements in the resulting matrices.

Solution

Rows: Row 1 – Farm 1; Row 2 –Farm 2. Columns: Column 1 – grain; Column 2 – milk; Column 3 – meat.

a) Total volume of products produced over the two years (2023 and 2024):

$$A_{total} = A_{2023} + A_{2024} = \begin{pmatrix} 2053 & 571 & 340 \\ 2679 & 453 & 694 \end{pmatrix} + \begin{pmatrix} 2745 & 644 & 800 \\ 2900 & 451 & 373 \end{pmatrix} = \begin{pmatrix} 4798 & 1215 & 1140 \\ 5579 & 904 & 1067 \end{pmatrix}$$

Economic meaning of the resulting matrix:

Total production matrix shows the combined volume of grain, milk, and meat produced by each farm over two years. For example, Farm 1 produced 4798 units of grain in total.

b) Growth in production volumes in 2024 compared to 2023:

$$A_{growth} = A_{2024} - A_{2023} = \begin{pmatrix} 2745 & 644 & 800 \\ 2900 & 451 & 373 \end{pmatrix} - \begin{pmatrix} 2053 & 571 & 340 \\ 2679 & 453 & 694 \end{pmatrix} = \begin{pmatrix} 692 & 73 & 460 \\ 221 & -2 & -321 \end{pmatrix}$$

Economic meaning of the resulting matrix:

Growth matrix shows how each product's output changed from 2023 to 2024. For example, Farm 1 increased meat production by 460 units, while Farm 2 decreased meat production by 321 units.

Guided Inquiry (Middle level – teacher sets problem and provides resources; students devise own procedures) *(Picture 4)*

Topic: Matrix equations and inverse matrices

Students are just given a matrix equation task in our book, it is problem 10, exercise 27. Teachers just provide the equation and the two matrices, the concept of the inverse matrix, and the formula for solving the exercise. The teacher's role is only to support students in checking their calculations and interpreting the results, not to give students a step-by-step guide. The outcome should be that students independently find the solution.

Picture 4 *is adapted from* a *linear algebra textbook, using matrices with the Leontief input-output model*

Problem 5

A company has three workshops, each producing a different type of product: workshop 1 produces product type 1; workshop 2 produces product type 2; workshop 3 produces product type 3. Part of the output is used for internal consumption, while the rest is the final product delivered outside the system.

Tasks:

1. Determine the distribution of products among workshops used for internal consumption.
2. Find the total output of each workshop, given:

The input coefficient matrix (direct consumption matrix):

$$A = \begin{pmatrix} 0.6 & 0.2 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.1 & 0.7 \end{pmatrix}$$

The final demand vector:

$$Y = \begin{pmatrix} 120 \\ 90 \\ 60 \end{pmatrix}$$

Open Inquiry (Highest level – students select a problem, set a question, plan and execute entirely on their own) *(Picture 5)*

Topic: Using vectors in navigation (real-world, not curriculum-bound)
The teacher here does not help; they just look later to see if the answer is good. This IBL level is. This traditional problem gives all the data. To turn into an open inquiry, students would select their own ports, find real current wind data, and formulate their own vector model. In the future, no single problem number applies directly, but the navigation provides a template.

Picture 5 *is adapted from vectors text book, using vectors in navigation*

53. An aircraft takes off from an airport and flies 500 km on a bearing of 045°. It then changes course and flies 1200 km on a bearing of 105°. Finally, it changes course again and flies 1000 km nautical miles on a bearing of 210°.

Determine:
a) the aircraft's resultant displacement vector (in north–east components),
b) the distance from the airport to the aircraft,
c) the bearing of the aircraft's final position from the airport.

The three inquiry-based learning levels (structured, guided, and open) represent a range of student autonomy and teacher engagement. As shown in Figure 4, the transition from organized to open inquiry progressively shifts accountability from the teacher to the students, enabling them to grow in independence, critical thinking, and problem-solving abilities at their own speed.

The difficulty of the mathematical subject, the students' age and cognitive development, their prior experience with inquiry methods, and the particular learning aims of the lesson all play a role in choosing an acceptable IBL level. An organized method offers the required scaffolding and boosts confidence for beginning students or when presenting new mathematical topics. Teachers can gradually transition to guided and eventually open inquiry, where students accept complete responsibility for their learning, as students gain procedural fluency and grow more adapted to inquiry processes. Best educational results are attained at the organized and guided levels, when the right kind of help is given without taking away the challenge of discovery. Even if it requires more of students, open inquiry provides excellent chances for creativity, in-depth engagement, and the growth of research skills, especially when it is related to subjects that students are truly interested in.

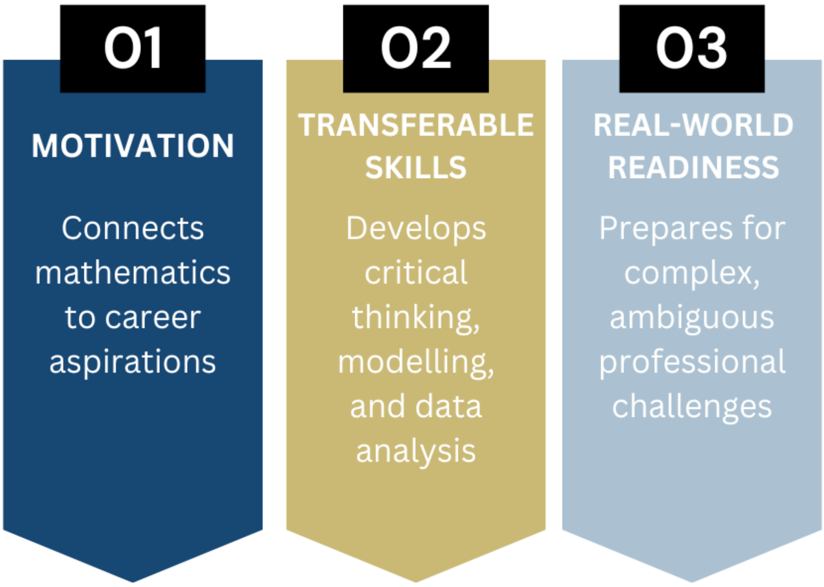
Real-life relevance

Students frequently view mathematics as an abstract and unconnected subject, with formulas, rules, and procedures that seem to have little bearing on the real world outside of the classroom. Probably have heard how students say “I am 21 years old and still not one day for use of cos or sin”. Usually, low motivation, reduced engagement, and a lack of awareness of the importance of mathematical knowledge in everyday and professional contexts are all consequences of this perspective. Therefore, bridging the gap between theoretical ideas and their practical applications, making mathematics relevant, meaningful, and applicable to real-world situations, is one of the main challenges in teaching mathematics in higher education.

In order to address this issue, the project "Traditions and Innovations in Baltic and Nordic Mathematics Education/2025" incorporates real-world relevance into its teaching strategies and resources. The educational materials created within this guidebook are intended to show how mathematics is an effective instrument for analyzing, modelling, and resolving issues that arise in both professional practice and society at large, rather than just teaching mathematical knowledge. The materials seek to demonstrate to students that mathematics is not a stand-alone field of knowledge but rather a crucial part of creativity, decision-making, and problem-solving in the modern world by relating mathematical ideas to real-world situations from engineering, technology, business, navigation, environmental systems, and other STEM-related fields.

There are several uses for real-world relevance in mathematics teaching (See Figure 6). First, it increases student motivation by providing a response to the frequently asked question, "Why do I need to learn this?" Students are more likely to connect deeply with the topic when they observe how mathematical ideas are applied in careers they hope to pursue. Second, it fosters transferable abilities like critical thinking, data analysis, modelling, and quantitative reasoning, which companies in various industries are increasingly looking for. Third, it helps students get ready for the challenges of the working world, where mathematical issues are rarely neatly packaged textbook exercises but instead call on the capacity to comprehend, adapt, and apply mathematical knowledge to new and frequently ambiguous situations

Figure 6 Real-world relevance



Innovation, technology, and sustainable development are all highly valued in the Baltic and Nordic regions, and they all strongly depend on mathematical skills. The economic development of these areas depends on industries like data science, engineering, renewable energy, information technology, and maritime logistics. These fields need graduates who can model complicated systems, reason mathematically, and make data-driven judgements. This project helps create a workforce that is not just mathematically literate but also capable of using mathematical reasoning to solve practical problems by coordinating mathematics instruction with the demands of society and the job market.

To show how mathematical ideas from linear algebra, calculus, vectors, and other fields can be placed into real-world professional and everyday situations, specific examples of real-life application problems are provided in the following section that are included in our book. The project's objective of making mathematics education more interesting, meaningful, and connected to the world outside of the classroom is eventually supported by these examples, which highlight the usefulness of mathematics and give teachers guidance for creating their own relevance-oriented assignments.

Example 1

To teach students about vectors, we do not just use theory; we use real life resemble for example, using our hands to explain "Vector or Cross product of two vectors".

Picture 6 *example from vectors text book*

A Right-hand triple $\vec{a}, \vec{b}, \vec{c}$ can also be visualized using the right-hand rule:

If you place the index finger of your right hand along the first vector $\vec{a}$ and the middle finger along the second vector $\vec{b}$, then your thumb points in the direction of the third vector $\vec{c}$.

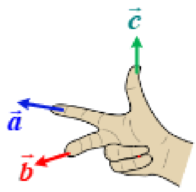


Figure 32. The direction of the cross product
(image adapted from work by Acdx licensed under CC-BY-SA 3.0)

Another explanation of the same rule:

If vectors $\vec{a}, \vec{b}, \vec{c}$ form a right-handed triple, you can determine the direction of $\vec{c}$ by placing the fingers of your right hand along the first vector $\vec{a}$ and bending them toward the second vector $\vec{b}$. Your thumb will point in the direction of the third vector $\vec{c}$.

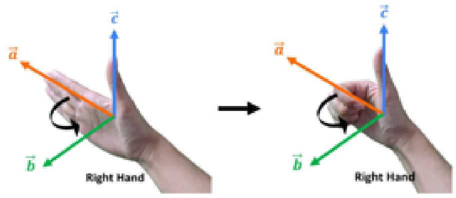


Figure 33. The direction of the cross product
(image from https://www.wizeprep.com/online-courses)

Example 2

Vectors are used in mechanics, mechanical engineering, and robotics to examine forces acting on objects, compute structural loads (such as those on buildings, bridges, and machine parts), and investigate the kinematics of mechanisms and the motion of rigid bodies. They serve as the mathematical cornerstone of dynamics, kinematics, and statics and are crucial to vibration theory, stress analysis, and dynamic system modelling.

Picture 7 *example from vectors text book*

A medium-class 6-axis industrial robot holds a payload of mass $m = 10$ kg. A fixed Cartesian coordinate system is attached to the robot base and is shown in Fig.49.

The center of the robot's elbow joint is located at point $B(0.2, 0.25, 0.7)\ m$.

The tool flange (end-effector mounting point) is located at point $E(0.5, 0.6, 0.9)m$.

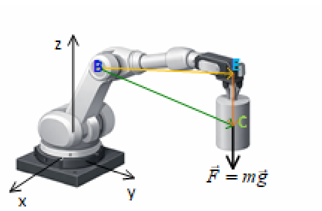


Figure 49. An industrial robot holds a payload

The payload is rigidly grasped. Its center of mass C is located $d = 0.12$m below the tool flange along the negative Z-axis.

Find the moment of the force (torque) of the payload weight about the elbow joint point B and its magnitude.

Assumptions: Static analysis. Only the payload weight is considered; the self-weight of the robot links (arm/forearm) and the gripper is neglected.

Example 3 (A similar formula is used in GPS navigation to calculate an object's current velocity, determine its direction of motion (for example, a car), and predict its future position.)

Picture 8 *example from vectors text book*

For example, to model light reflection from a matte or rough surface (such as concrete, paper, or plaster), the diffuse lighting model is used, in which light is reflected uniformly in all directions.

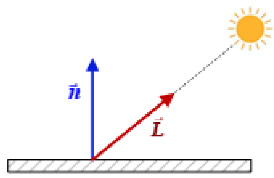


Figure 58(a).

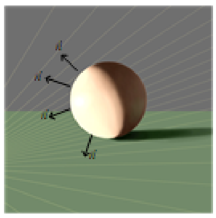


Figure 58(b). Lighting in 3D graphics

This model is described by Lambert's reflection model, according to which the brightness of an illuminated surface depends only on the angle of incidence of the light (Fig.58). The amount of light reaching the surface is given by the following formula:

$$I = k_d\, I_L\, \vec{L} \cdot \vec{n}$$

Example 4
We also made book about liner algebra and here are some examples of tasks that used real life relevance.
Picture 9 *example from linear algebra text book*

A European manufacturer plans to produce the following number of units in September: 600 electric kettles; 950 toasters and 400 microwave ovens.

To manufacture these appliances, the company uses five types of materials: steel, plastic, glass, copper wire, and insulation. The required amount of each material (in kg per unit) and the cost per kilogram in euros are provided below.

Material requirements per unit (kg):

Appliance	Steel	Plastic	Glass	Copper wire	Insulation
Electric kettles	1.2	0.8	0.4	0.2	0.1
Toasters	1.0	1.1	0.2	0.3	0.2
Microwave ovens	2.5	1.8	1.0	0.5	0.3

Cost per kilogram of materials (in euros): Steel: €3.20; Plastic: €2.40; Glass: €1.80; Copper wire: €6.00 and Insulation: €2.00.

Tasks:

a) Calculate the total quantity of each material needed for the entire production plan.
b) Determine the material cost per unit of each appliance.
c) Find the total material cost for producing all planned units.

Example 5 this example shows how using matrix you can calculate data and real life based situation to knows how many buses the company will need.
Picture 10 *example from linear algebra text book*

8) A transportation company has three bus routes (rows) and data for four days (columns):

$$A = \begin{pmatrix} 120 & 135 & 150 & 140 \\ 80 & 95 & 100 & 110 \\ 200 & 210 & 190 & 205 \end{pmatrix}$$

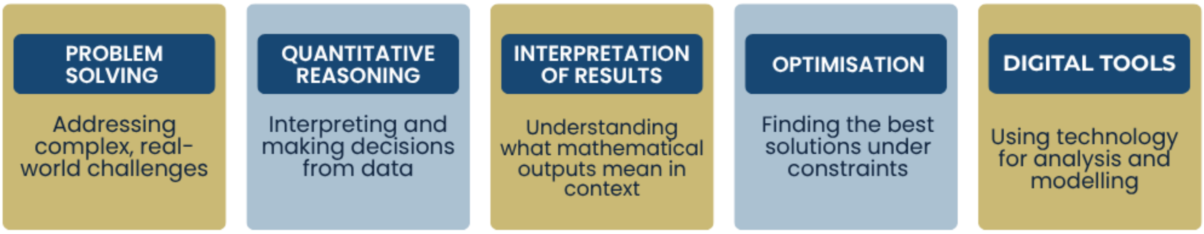
Find the total number of passengers over 4 days for each route and the daily average. Model the effect of a 15% increase in passenger traffic (e.g., adding one more bus) and recalculate the totals.

Labor market relevance

In Åland, Estonia, and Latvia, mathematics forms an essential foundation for studies and professions in engineering, technology, business, data analysis, maritime industries, and other STEM-related fields. As workplaces increasingly rely on data, digital technologies, modelling, and analytical decision-making, graduates require mathematical competencies that extend beyond procedural knowledge and support the application of mathematics in professional contexts.

From a labor market perspective, teaching materials should help students understand mathematics as a practical tool for analyzing, modelling, and solving real-world problems. Competencies such as problem-solving, quantitative reasoning, interpretation of results, optimization, and the use of digital tools are relevant across a wide range of professions and contribute to graduates' ability to adapt to changing workplace requirements (See Figure 7).

Figure 7 Labor market competencies



Teacher guidelines can strengthen this connection by encouraging the use of professional and sector-specific examples relevant to the Baltic and Nordic regions. These may include applications from engineering, ICT, business, navigation, maritime operations, environmental systems, and other contexts where mathematical thinking supports decision-making and innovation.

By making the practical relevance of mathematics more visible, teaching materials can help students recognize how mathematical knowledge transfers to professional practice and societal challenges. This contributes to the development of competencies valued in the labor markets of Åland, Estonia, and Latvia, while supporting the broader goal of strengthening STEM education and graduate preparedness for working life.

Evidence from research: the labour market value of basic skills

Basic math skills are substantially linked to improved labor market outcomes, according to research. Students who fall short of baseline mathematics proficiency requirements are less likely pursue higher education and have less advantageous career paths (Meehan, Pacheco, & Schober, 2023).[19]

The likelihood that young people with poor mathematical skills will pursue further education is significantly lower. According to Meehan et al. (2023)[19], individuals with lower skill levels are more than four times less likely to finish a bachelor's degree and nearly five times less likely to gain university entrance. Higher education is a crucial pathway to better employment opportunities and higher incomes; therefore, this educational inequality has long-term consequences. This disparity is not merely an individual problem but reflects what Müller[20] (2025) describes as the "socio-economic valuation gap" – a structural tension where market incentives reward technical proficiency while marginalizing the critical competencies needed for democratic accountability and sustainable development.

Research also shows that throughout their lives, young people with low mathematics skills have lower employment rates. Men with lower skill levels may initially earn more than their higher-skilled counterparts when they are younger, often because they start working full-time earlier while their more skilled peers continue their education. However, by their early twenties, this trend reverses. Men with low skills earn about 15% less than men with higher skills by the time they are 25 years old. By the age of 25, women with lower skill levels earn almost 35% less than those with higher ability levels (Meehan et al., 2023[19]). This gender disparity in earnings is significant, as women with low skills face more than double the earnings penalty of men in the same position.

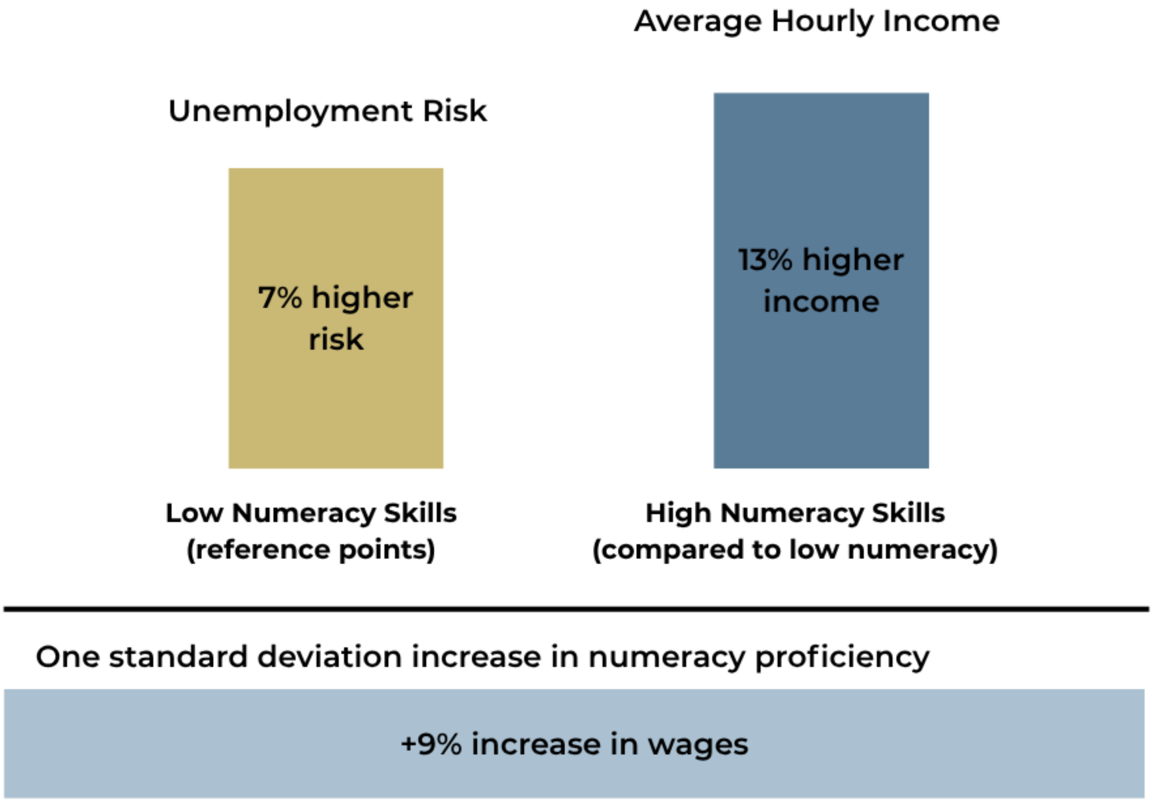
While young people with higher mathematics skills are more likely to work as professionals and in clerical or administrative roles, those with lower mathematics skills are more likely to engage in manual occupations such as laborers and machinery operators. These occupational differences reflect the types of roles that require proficiency in the kind of mathematical thinking assessed in skills evaluations. The earnings gap between skill groups is partly explained by this occupational segregation (Meehan et al., 2023[19]). Müller[20] (2025) argues that this pattern is not accidental but reflects a deeper process of "mathematization" – where mathematical logic becomes the predominant guidebook for ordering economic life, creating winners and losers based on mathematical competence.

Beyond employment and earnings, the consequences of low mathematics skills extend to broader life outcomes. Müller[20] (2025) identifies several socio-economic tensions arising from the increasing mathematization of society: the "enclosure of imagination" – where mathematical optimization logic restricts the capacity to imagine sustainable alternatives – and the "material opaqueness" of mathematics, where its negative social and environmental impacts are often hidden from those who benefit from it. The erosion of multilateralism further compounds these challenges, undermining the global coordination essential for regulating mathematical technologies and addressing sustainability imperatives (Müller,

[19] Meehan, L., Pacheco, G., & Schober, T. (2023). *Basic reading and mathematics skills and the labour market outcomes of young people: Evidence from PISA and linked administrative data* (Economics Working Paper Series No. 2023/01). Auckland University of Technology.
[20] Müller, D. (2025). *The mathematisation of the world: Uncovering the socio-economic tensions for ethics in mathematics education* (Working Paper). Auckland University of Technology. https://doi.org/10.48550/arXiv.2510.23349

2025)[20]. OECD 2024[21]analysis confirms that adults with higher numeracy skills are more likely to be employed, earn a higher wage, and report better health and life satisfaction than those with lower numeracy skills. Specifically, see in Figure 8 on how low numeracy skills are associated with an increase in an individual's risk of being unemployed or inactive by around 7%, while high numeracy skills are associated with an average hourly income 13% greater than low numeracy skills. On average across OECD[21] countries, a one-standard-deviation increase in numeracy proficiency is associated with a 9% increase in wages

Figure 8: Impact of numeracy skills on labor market outcomes.



Jonas (2018)[22] research shows that financial literacy, which affects choices that have a significant impact on people's lives, such figuring out mortgage repayments, tax returns, and pension investments, is likewise shaped by mathematics. Mathematics has an impact on people's decision-making abilities and health outcomes. Compared to adults with low numeracy, those with strong numeracy are 65% less likely to report poor health.

Müller (2025)[20] lists a number of socioeconomic conflicts brought on by society's growing mathematization: the "material opaqueness" of mathematics, where its detrimental effects on society and the environment are frequently concealed from those who profit from it, and the "enclosure of imagination"—where mathematical optimization logic limits the ability to envision sustainable alternatives. These difficulties are made worse by the decline of multilateralism, which threatens the international cooperation necessary to control mathematical technology and meet sustainability requirements.

[21] OECD. (2024). *Education at a glance 2024: OECD indicators*. OECD Publishing. https://doi.org/10.1787/c00cad36-en.
[22] Jonas, N. (2018), "Numeracy practices and numeracy skills among adults", *OECD Education Working Papers*, No. 177, OECD Publishing, Paris, https://doi.org/10.1787/8f19fc9f-en.

From an educational perspective, these findings underscore the importance of equipping students not only with mathematical skills but also with the critical awareness to understand how mathematics functions in society, who benefits from it, and who is left behind. As Müller (2025)[20] argues, mathematics education must move beyond purely technical training to prepare students for civic life in an increasingly mathematical world, where the ability to question, critique, and imagine alternatives is as important as procedural fluency.

Based on the evidence presented, the guidebook developed in this project addresses labour market needs (see table 9)

Table 9
Labor market needs

Need	How the Guidebook Addresses It
Real-world application	Tasks connect mathematical concepts to professional contexts
Quantitative reasoning	Students develop data analysis and interpretation skills
Problem-solving	Inquiry-based tasks develop critical thinking
Digital tool use	Integration of software (GeoGebra, MATLAB, etc.)
Critical awareness	Understanding how mathematics functions in society

Sustainable Mathematics Education

The rapid development of contemporary society, technological progress, and transformations in the labour market are increasingly amplifying the demand for specialists in STEM fields. Concurrently, growing attention is being directed towards sustainable development, which encompasses not only environmental but also educational and social dimensions. The United Nations 2030 Sustainable Development Goals (2015)[23] emphasise the necessity for quality education that prepares students for contemporary challenges, including technology management and sustainable thinking. In this context, higher-education mathematics plays a key role by laying the foundation for critical thinking, problem-solving, and analytical skills.

Sustainable mathematics education refers to teaching, learning, and assessment approaches that ensure relevance, effectiveness, and environmental responsibility (Joutsenlahti & Perkkilä, 2019)[24]. It encompasses the following components:

Table 10
Characterization of Elements of Sustainable Mathematics Education

Element of Sustainable Mathematics Education	Description
Real-world problem solving	Mathematical tasks related to resource management, environmental pollution, climate change, and other pertinent issues
Interdisciplinary approach	Integration of mathematics with other disciplines to address complex problems
Technology integration	Utilisation of modern technologies in the learning process to enhance understanding and accessibility
Critical thinking	The capacity to analyse, evaluate, and make informed decisions based on mathematical data
Ethical responsibility	Awareness of the long-term impact of mathematical decisions on the environment and society

Mathematics education is being reconfigured to foster sustainable thinking and practice, rendering both students and educators cognisant of the global situation (Widiati & Juandi, 2019)[25]. This implies that the content and methodologies of mathematics instruction are being adapted to promote not only mathematical proficiency but also the development of responsibility and sustainable action.

Research indicates that technology-based teaching methods are essential for ensuring sustainable mathematics education, particularly in the era of the Fourth Industrial Revolution (4IR) following the COVID-19 pandemic (Naidoo & Singh-Pillay, 2023). Technology-based teaching methods encompass

[23] United Nations. (2015). *Transforming our world: The 2030 Agenda for Sustainable Development*. https://sdgs.un.org/2030agenda
[24] Joutsenlahti, J., & Perkkilä, P. (2019). Sustainability development in mathematics education: A case study of what kind of meanings do prospective class teachers find for the mathematical symbol "2/3"? *Sustainability*, *11*(2), Article 457. https://doi.org/10.3390/su11020457
[25] Widiati, I., & Juandi, D. (2019). Philosophy of mathematics education for sustainable development. *Journal of Physics: Conference Series*, *1157*, Article 022128. https://doi.org/10.1088/1742-6596/1157/2/022128

the use of various digital tools, platforms, and devices for instruction, assessment, and the learning process. The Community of Inquiry (CoI) theoretical framework and the SAMR (Substitution, Augmentation, Modification, Redefinition) model (Naidoo & Singh-Pillay, 2023, p. 25)[26] provide valuable conceptual foundations. The CoI model underscores the importance of three interrelated components:

- **Social presence** – the capacity to communicate and interact in a safe online environment.
- **Teaching presence** – the instructor's support, guidance, and organisation of the learning process.
- **Cognitive presence** – the ability to construct meaningful knowledge through collaboration and problem-solving.

The SAMR model, in turn, facilitates an understanding of how technology can enhance or transform traditional pedagogy (see Table 11).

Table 11
Characterisation of SAMR Levels

SAMR Level	Description	Example in Mathematics
Substitution	Technology replaces traditional tools with no significant functional change	Digital textbook in place of a paper one
Augmentation	Technology enhances the learning process	Interactive tasks with immediate feedback
Modification	Technology allows for significant redesign of learning activities	Online collaboration to solve complex mathematical projects
Redefinition	Technology enables entirely new learning possibilities	Global collaboration in data analysis and mathematical modelling

To ensure sustainable mathematics education in higher education institutions, it is essential to consider several practical aspects derived from research and examples of good practice.

1. Resource Availability and the Digital Divide
One of the primary challenges is ensuring equitable access to technology and internet connectivity. Students from less advantaged backgrounds may lack the necessary digital resources, by this means intensifying educational inequality (Naidoo & Singh-Pillay, 2023, p. 25). Consequently, higher education institutions should promote the use of Open Educational Resources (OER) and provide technical support.

2. Meaningful Learning and Connection to Real Life
Sustainable mathematics education requires that the curriculum be linked to real-life problems and sustainable development goals (Joutsenlahti & Perkkilä, 2019, p. 23)[24]. For instance, students can analyse climate change data, model resource consumption, or develop optimisation solutions for renewable energy. Such tasks render mathematics meaningful and demonstrate its societal relevance.

3. Collaboration and Interdisciplinary

[26] Naidoo, J., & Singh-Pillay, A. (2023). Embedding sustainable mathematics higher education in the Fourth Industrial Revolution era post-COVID-19: Exploring technology-based teaching methods. *Sustainability*, *15*(12), Article 9692. https://doi.org/10.3390/su15129692

Sustainable development issues are complex and necessitate an interdisciplinary approach. Mathematics education should be integrated with other fields, such as engineering, economics, and environmental sciences, to enable students to address real-world problems. Collaboration among different faculties and institutions, as well as international cooperation (as exemplified in this Nordplus project), facilitates the exchange of experience and the development of innovative solutions.

In conclusion, it must be emphasised that sustainable mathematics education in higher education is not reducible solely to the acquisition of knowledge and skills within a narrow disciplinary understanding. Rather, it encompasses a broader perspective on the role of mathematics in society, professional practice, and sustainable development processes, accentuating accessibility, meaningful learning, interdisciplinary, and responsible technology use. Such an approach strengthens students' capacity not only to apply mathematical concepts and methods but also to critically evaluate their applications, recognise the connections between mathematics and real-world problems, and utilise acquired knowledge for the benefit of sustainable societal development.

Summary and conclusions

The project "Traditions and Innovations in Baltic and Nordic Mathematics Education/2025" addresses the critical need for improved mathematics education in the Baltic and Nordic regions. The European Union faces a significant shortage of skilled professionals in STEM fields, with Latvia having one of the lowest shares of STEM graduates in the EU (19.7% compared to the EU average of 26.6%). University mathematics teaching materials are often outdated, fragmented, and disconnected from the dynamic demands of the modern labor market and professional practice.

To address these challenges, the project brings together four higher education institutions from Latvia, Estonia, and the Åland Islands to collaboratively develop innovative teaching materials and methodologies that bridge the gap between theoretical mathematics education and its practical, real-world applications.

Key contributions of the guidebook

Inquiry-based mathematics education (IBME)
The guidebook adopts inquiry-based mathematics education as its core pedagogical approach. IBME encourages students to work in ways similar to mathematicians and scientists—observing phenomena, asking questions, formulating hypotheses, conducting investigations, analyzing data, and communicating results. This student-centered approach shifts the teacher's role from transmitter of knowledge to facilitator of learning. The Three-layer model of inquiry provides the theoretical foundation (see Table 12).

Table 12
The Three-layer model of inquiry

Layer	Focus	Key Activity
Layer 1	Students	Engaging with mathematical concepts through exploration and discovery
Layer 2	Teachers	Reflecting on teaching practices and designing inquiry-based tasks
Layer 3	Researchers	Analysing data and generating new knowledge to inform practice

The guidebook identifies three levels of inquiry that can be implemented progressively (see Table 13).

Table 13
Levels of inquiry-based learning

Level	Teacher Role	Student Role
Structured	Provides problem, procedures, and resources	Follows directions; discovers outcome
Guided	Sets problem and provides resources	Devises own procedures
Open	Sets initial context; minimal guidance	Selects problem, sets question, executes independently

Nordplus

TRADITIONS & INNOVATIONS IN MATHEMATICS EDUCATION
BALTIC & NORDIC EXPERIENCE

The guidebook connects mathematical concepts to real-world contexts across multiple fields (see Table 14).

Table 14
Real-life relevance

Field	Mathematical Concepts	Applications
Business and Manufacturing	Matrix operations, systems of equations	Revenue calculation, production planning, cost analysis
Engineering and Robotics	Vectors, cross products, linear systems	Circuit analysis, joint torque calculation, structural analysis
Navigation and GPS	Vector addition, bearings	Ground speed calculation, course planning, position tracking
Computer Graphics	Matrix transformations, dot products	3D modelling, image processing, lighting calculations
Image Processing	Matrix operations	Colour transformation, brightness adjustment

Labor market relevance
Evidence from research demonstrates that mathematical skills are substantially linked to improved labour market outcomes:

- Low numeracy skills are associated with a **7% higher risk** of unemployment or inactivity;
- High numeracy skills are associated with **13% higher** average hourly income;
- A one-standard-deviation increase in numeracy proficiency is associated with a **9% increase** in wages (OECD, 2024)[21];
- Adults with high numeracy are **65% less likely** to report poor health than those with low numeracy (Jonas, 2018)[22];
- Individuals with lower skill levels are more than **four times less likely** to finish a bachelor's degree (Meehan et al., 2023)[19].

These findings underscore the importance of equipping students not only with procedural mathematical skills but also with the critical awareness to apply mathematics in professional and everyday contexts.

Conclusions

The project "Traditions and Innovations in Baltic and Nordic Mathematics Education/2025" makes a significant contribution to improving mathematics education in higher education across the Baltic and Nordic regions. By implementing inquiry-based mathematics education through Communities of Inquiry, the project achieves the following:

1. **Bridge the gap** between theoretical mathematics and practical applications through real-world examples and inquiry-based tasks.
2. **Enhance teacher professional development** through collaborative inquiry, reflection, and the sharing of best practices across partner institutions.
3. **Improve student engagement and conceptual understanding** by moving beyond procedural learning to develop deeper, relational understanding of mathematical concepts.
4. **Prepare graduates for STEM careers** by developing competencies valued in the labour market, such as problem-solving, quantitative reasoning, and the use of digital tools.
5. **Support inclusive education** for diverse learners by providing teaching materials in English and incorporating principles of accessible design.
6. **Strengthen regional collaboration** by establishing sustainable partnerships between higher education institutions in Latvia, Estonia, and Finland.

The three-layer model of inquiry provides a comprehensive guidebook for achieving these goals, with activities at the levels of student inquiry, teacher inquiry, and developmental research inquiry. The guidebook developed in this project contains modern teaching resources, real-life application tasks, and examples at different levels of inquiry-providing educators with practical tools to transform mathematics teaching and learning.

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